Analysis

Paper 2 of Maths is slightly tough than the Paper 1. and also it contain negative marking in whole paper There are many question from our classroom notes.

Permutation-Combination & Probability → 4 que.
Full Trigonometry → 2 que
2D geometry → 3 que.
Diff. & Integral Calculus → 6 que.
Binomial Theorem → 1 ques.
Basic Mixture Match Matrix → 3 que
Complex Number 1 Match Matrix → 1 que

20 que

Thanks
SUHAG

P.T.O

Code'0'/single Correct type +3/-1

Q.41. Six Cards and six envelopes are numbered 1, 2, 3, 4, 5, 6 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and more over the card numbered 1 is always placed in envelope numbered 2. Then the number of ways it can be done is A) 264 B) 265 C) 53 D) 67

Sol. $\boxed{1}$ $\underset{①}{\boxed{2}}$ $\boxed{3}$ $\boxed{4}$ $\boxed{5}$ $\boxed{6}$

If ② set in $\boxed{1}$ then dearrangment of only 4 items $\lfloor 4\left(\frac{1}{\lfloor 0}-\frac{1}{\lfloor 1}+\frac{1}{\lfloor 2}-\frac{1}{\lfloor 3}+\frac{1}{\lfloor 4}\right)=9$

If ② not set in $\boxed{1}$ then dearangment of 5 itemes $\lfloor 5\left(\frac{1}{\lfloor 0}-\frac{1}{\lfloor 1}+\frac{1}{\lfloor 2}-\frac{1}{\lfloor 3}+\frac{1}{\lfloor 4}-\frac{1}{\lfloor 5}\right)$ $=44$

So Ans $9+44=53$

Apply dearrangment for all 6 card at diffent location. (Total Possibilities)

${}^6C_6 \cdot D_6$

$1\times\lfloor 6\left[\frac{1}{\lfloor 0}-\frac{1}{\lfloor 1}+\frac{1}{\lfloor 2}-\frac{1}{\lfloor 3}+\frac{1}{\lfloor 4}-\frac{1}{\lfloor 5}+\frac{1}{\lfloor 6}\right]$

$720\left[1-1+\frac{1}{2}-\frac{1}{6}+\frac{1}{24}-\frac{1}{120}-\frac{1}{720}\right]$

$0+360-120+30-6-1$

263 are those ways in which card 1 can put any of envelope 2,3,4,5,6.

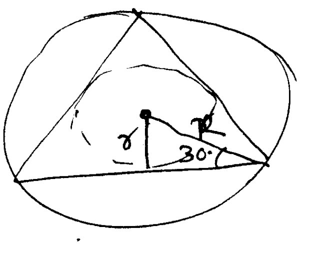
Q42 In a triangle the sum of two sides is x and the product of the same two sides is y. If $x^2-c^2=y$ where c is the third side of the triangle then the ratio of the inradius to the circumradius of the triangle is A) $\frac{3y}{2x(x+c)}$ B) $\frac{3y}{2c(x+c)}$ C) $\frac{3y}{4x(x+c)}$ D) $\frac{3y}{4c(x+c)}$

Sol. Consider equilateral Δ with sides 1, 1, 1

So $x=2,\ y=1,\ c=1$

Ⓑ

$\frac{r}{R}=\sin 30^\circ=\frac{1}{2}$

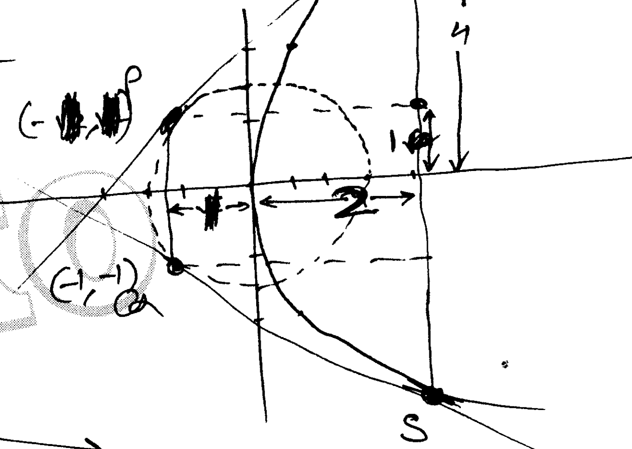


Check Options Ⓑ ✓

P.T.O.

SOLUTION OF JEE ADVANCED 2014, PAPER 2 WITH IN 45 MIN., PAGE 2(b) OF 10

Q.43. The common tangents to the circle $x^2+y^2=2$ and the parabola $y^2=8x$ touch the circle at the points P, Q and the parabola at the point R, S. Then the area of the quadilateral PQRS A) 3 B) 6 C) 9 D) 15



Sol. (D) (Ans 15)

$$y^2 = 4ax \rightarrow y^2 = 4 \cdot 2 \cdot x \rightarrow a = 2$$

$$T \quad y = mx + \frac{a}{m}$$

$$y = mx + \frac{2}{m}$$

Now $x^2 + y^2 = r^2 \rightarrow r^2 = 2$

$$y = mx \pm r\sqrt{1+m^2}$$

$$y = mx \pm \sqrt{2}\sqrt{1+m^2}$$

common $\frac{2}{m} = \sqrt{2}\sqrt{1+m^2}$

$$^{2}\frac{\cancel{4}}{m^2} = \cancel{2}(1+m^2)$$

$$2 = m^2(1+m^2)$$

$$m^2 = 1$$

$M = 1$ P(-[illegible], [illegible])

& R(2, 4)

$$\frac{dy}{dx} = 1$$

$$2y\frac{dy}{dx} = 8$$

$$1 = \frac{8}{2y}$$

$$y = 4 \rightarrow y^2 = 8x$$

$$16 = 8x$$

$$2 = x$$

Req area

~~2[[illegible]]~~

$$2\left[(3 \times 1) + \frac{1}{2}(3 \times 3)\right]$$

~~2[[illegible]] = 6~~

$$2\left[3 + \frac{9}{2}\right] = 6 + 9 = 15$$

Q.44. Three boys and two girls stand in a queue. The probability, that the number of boys ahead of every girl is at least one more than the number of girls ahead of her, is (A) $\frac{1}{2}$, (B) $\frac{1}{3}$, (C) $\frac{2}{3}$, (D) $\frac{3}{4}$

(A)

Sol. Total ways $\rightarrow \lfloor 5$

Different Cases BBBGG, BBGBG, BGBBG, BBGGB, BGBGB
5

$$\frac{5(\lfloor 3\,\lfloor 2)}{\lfloor 5} = \frac{5.\cancel{6}.2}{\cancel{120}_{10}} = \boxed{\frac{1}{2}\text{ Ans}}$$

P.T.O.

Q.45. The quadratic equation $p(x)=0$, with real coefficients has purely imaginary roots. then the equation $p(p(x))=0$ has

A) Only purly imaginary roots B) all real roots

C) two real and two purely imaginary roots

D) neither real nor purely imaginary roots.

Sol. Consider $p(x) = 1x^2 + 0x + 1 = x^2 + 1$

(D)

$$x^2+1=0$$
$$x^2=-1$$
$$x = \pm\sqrt{-1} \begin{cases} -i \\ +i \end{cases}$$

$$P(P(x)) = (P(x))^2+1$$
$$=(x^2+1)^2+1$$
$$=x^2+2x+1+1=0$$
$$x^2+2x+2=0$$
$$D = 2^2-4\cdot1\cdot2$$
$$=-ve$$

Q.46. for $x \in (0,\pi)$, the equation $\sin x + 2\sin 2x - \sin 3x = 3$ has

A) infinitely many solution B) three sol.

C) one sol. D) No sol.

Sol. Class room question

Ans (D) No Sol.

Q.47. The following integral $\int_{\pi/4}^{\pi/2} (2\operatorname{cosec} x)^{17} dx =$

(A)

A) $\int_0^{\log(1+\sqrt{2})} 2(e^u+e^{-u})^{16} du$ B) $\int_0^{\log(1+\sqrt{2})} (e^u+e^{-u})^{17} du$

C) $\int_0^{\log(1+\sqrt{2})} (e^u-e^{-u})^{17} du$ D) $\int_0^{\log(1+\sqrt{2})} 2(e^u-e^{-u})^{16} du$

Sol. Put $\ln\tan\frac{x}{2} = t$

$$\tan\frac{x}{2} = e^t$$
$$\sin x = \frac{2e^t}{1+e^{2t}}$$
$$\operatorname{cosec} x = \frac{e^t+e^{-t}}{2}$$

$$I = 2\int_{\ln(\sqrt{2}-1)}^{0} (e^t+e^{-t})^{16} dt$$

So Ans is (A)

P.T.O.

Q.48. Coefficient of x^{11} in the expansion of

(C)

$$(1+x^2)^4(1+x^3)^7(1+x^4)^{12}$$

A) 1051 B) 1106 C) 1113 D) 1120

Sol. $\left({}^4C_0 + {}^4C_1x^2 + {}^4C_2x^4 + {}^4C_3x^6 + {}^4C_4x^8\right)\left({}^7C_0 + {}^7C_1x^1 + {}^7C_2x^6 + {}^7C_3x^9 + ----\right)$

$\left({}^{12}C_0 + {}^{12}C_1x^4 + {}^{12}C_2x^8 + ---\right)$

On calculation we will get Ans (C) 1113

Q.49. let $f:[0,2]\rightarrow R$ be a function which is continuous on $[0,2]$ and is differentiable on $(0,2)$ with $f(0)=1$. let

(B)

$F(x) = \int_0^{x^2} f(\sqrt{t})\,dt$ for all $x \in [0,2]$, If $F'(x)=f'(x)$ for all $x \in (0,2)$, then $F(2)$ equals to

A) e^2-1 B) e^4-1 C) $e-1$ D) e^4

So finally $f(2)=e^4-1$

Sol. $f'(x) = 2xf(x)$ [Newton lebnitz]

$\frac{f'(x)}{f(x)} = 2x$ on integration

$\rightarrow \ln(f(x)) = x^2+c$

$x=0,\ f(0)=1,\ c=0$

$\ln f(x) = x^2$

$f(x) = e^{x^2}$

$F(x) = f(x)+c = e^{x^2}+c'$

$F(0)=0;\ c'=-1$

Q.50. The function $y=f(x)$ is the solution of the

(B) differential equation $\frac{dy}{dx} + \frac{xy}{x^2-1} = \frac{x^4+2x}{\sqrt{1-x^2}}$

in $(-1,1)$ satisfying $f(0)=0$. Then $I = \int_{-\sqrt{3}/2}^{\sqrt{3}/2} f(x)\,dx$ is

A) $\frac{\pi}{3}-\frac{\sqrt{3}}{2}$ B) $\frac{\pi}{3}-\frac{\sqrt{3}}{4}$ C) $\frac{\pi}{6}-\frac{\sqrt{3}}{4}$ D) $\frac{\pi}{6}-\frac{\sqrt{3}}{2}$

Sol. I.F. $=\sqrt{1-x^2}$

so $y = \frac{\frac{x^5}{5}+x^2}{\sqrt{1-x^2}}$

$I = \int_{-\sqrt{3}/2}^{\sqrt{3}/2} \frac{\frac{x^5}{5}+x^2}{\sqrt{1-x^2}} = 2\int_0^{\sqrt{3}/2} \frac{x^2}{\sqrt{1-x^2}}$ let $\sin\theta = x$

$I = \frac{\pi}{3} - \frac{\sqrt{3}}{4}$

P.T.O.

Paragraph for que 51 & 52

Box 1 contains three cards bearing numbers 1, 2, 3; box 2 contains 5 cards bearing numbers 1, 2, 3, 4, 5 and box 3 contains seven cards 1, 2, 3, 4, 5, 6, 7. A card is drawn from each of the boxes. let x_i be the number on the card drawn from the i^{th} box, $i = 1, 2, 3$

Q.51 The probability that $x_1 + x_2 + x_3$ is odd, is

A) $\frac{29}{105}$ B) $\frac{53}{105}$ C) $\frac{57}{105}$ D) $\frac{1}{2}$

Sol. it is possible when all 3 odd or 2 even & 1 odd

$(O_1 O_2 O_3)$ or $(O_1 E_2 E_3)$ or $(E_1 O_2 E_3)$ or $(E_1 E_2 O_3)$

$$\frac{2}{3}\cdot\frac{3}{5}\cdot\frac{4}{7} + \frac{2}{3}\cdot\frac{2}{5}\cdot\frac{3}{7} + \frac{1}{3}\cdot\frac{3}{5}\cdot\frac{3}{7} + \frac{1}{3}\cdot\frac{2}{5}\cdot\frac{4}{7}$$

$$= \frac{24}{105} + \frac{12}{105} + \frac{9}{105} + \frac{8}{105} = \frac{53}{105}$$

Q.52. The ~~probability~~ probability that x_1, x_2, x_3 are in arithmetic progression, is A) $\frac{9}{105}$, B) $\frac{10}{105}$, C) $\frac{11}{105}$ D) $\frac{7}{105}$

Sol. (C)

AP rule अगलबगलका Total = बिचका Double

$$\underset{\text{even}}{2x_2} = x_1 + x_2 \xrightarrow{} x_1 \,\&\, x_2 \underbrace{\text{both even}}_{1\times 3 = 3\text{ ways}} \text{ or } \underbrace{\text{both odd}}_{2\times 4 = 8\text{ ways}}$$

$3 + 8 = 11$ ways

So Ans $\frac{11}{105}$

Paragraph for question 53 and 54

let a, r, s, t be non zero real numbers, let $P(at^2, 2at)$, Q, $R(ar^2, 2ar)$ and $S(as^2, 2as)$ be distinct points on the parabola $y^2 = 4ax$ Suppose PQ is the focal chord and the lines QR and PK are parallel, where K is the point $(2a, 0)$

P.T.O.

Q.53. The value of r is A) $-\frac{1}{t}$ B) $\frac{t^2+1}{t}$ C) $\frac{1}{t}$ D) $\frac{t^2-1}{t}$

Sol. Class room question

Ans (D)

$$r = \frac{t^2-1}{t}.$$

Q.54. If $st=1$, then the tangent at P and the normal at S to the parabola meet at a point whose ordinate is A) $\frac{(t^2+1)}{2t^3}$ B) $\frac{a(t^2+1)}{2t^3}$ C) $\frac{a(t^2+1)^2}{t^3}$ D) $\frac{a(t^2+2)^2}{t^3}$

Sol.: Class room qustn.

Ans (B)

$$y = \frac{a(t^2+1)}{2t^3}.$$

Paragraph for que 55 & 56

Given that for each $a \in (0,1)$ $\lim_{h\to 0^+} \int_h^{1-h} t^{-a} \cdot (1-t)^{a-1} \cdot dt$ exist. let this limit be $g(a)$

In addition, it is given that the function $g(a)$ is differentiable on $(0,1)$

Q55 The value of $g(1/2)$ is A) π B) 2π C) $\frac{\pi}{2}$ D) $\frac{\pi}{4}$

Sol. (A) $g'(a) = \int_h^{1-h} \frac{\partial}{\partial a} t^{-a}(1-t)^{a-1} dt = 0$

So $g(a) =$ constant. P.T.O.

Sol.55. from last Page

(A) let $g(a) = \cancel{\star}$

make perfect sq.

$$g(a) = \lim_{h\to 0^+}\int_h^{1-h}\frac{1}{\sqrt{t(1-t)}}\,dt = \left[\sin^{-1}\frac{(t-\frac{1}{2})}{(1/2)}\right]_h^{1-h}$$

$$= \pi$$

Q.56. The value of $g'(1/2)$ is A) $\frac{\pi}{2}$ B) π C) $-\frac{\pi}{2}$ D) 0

Sol. (D)

$$g(a) = \lim_{h\to 0^+}\int_h^{1-h} t^{-a}(1-t)^{a-1}\,dt$$

$$g(1-a) = \lim_{h\to 0^+}\int_h^{1-h} (\cancel{1-}t)^{a-1}(\cancel{t})^{1-a}$$

$$\longrightarrow g(a) = g(1-a)$$

$$g'(a) = -g'(1-a)$$

$$a = \frac{1}{2},\ -g'(\tfrac{1}{2}) = g'(\tfrac{1}{2}) \longrightarrow 0 = 2g'(\tfrac{1}{2})$$

$$0 = g'(\tfrac{1}{2})$$

Q 57. (D)

List I

P. The number of polynomials $f(x)$ with non-negative integer coefficients of degree ≤ 2, satisfying $f(0) = 0$ and $\int_0^1 f(x)dx = 1$, is

Q. The number of points in the interval $[-\sqrt{13}, \sqrt{13}]$ at which $f(x) = \sin(x^2) + \cos(x^2)$ attains its maximum value, is

R. $\int_{-2}^{2}\frac{3x^2}{(1+e^x)}\,dx$ equals

S. $\dfrac{\left(\int_{-\frac{1}{2}}^{\frac{1}{2}}\cos 2x\,\log\left(\frac{1+x}{1-x}\right)dx\right)}{\left(\int_0^{\frac{1}{2}}\cos 2x\,\log\left(\frac{1+x}{1-x}\right)dx\right)}$ equals

List II

1. 8

2. 2

3. 4

4. 0

	P	Q	R	S
(A)	3	2	4	1
(B)	2	3	4	1
(C)	3	2	1	4
(D)	2	3	1	4

$[-\sqrt{13}, \sqrt{13}]$

$\sqrt{2}\sin(x^2) + \cos(x^2)$

P.T.O.

Sol. 57. Starting with P

Ans D

Now work on

(R) $I = \int_{-2}^{2} \frac{3x^2}{1+e^{x}}$

$x \to -2+2-x \to -x$

$I = \int_{-2}^{2} \frac{3x^2}{1+\frac{1}{e^x}}$

$2I = \int_{-2}^{2} \frac{3x^2\cancel{(1+e^x)}}{\cancel{1+e^x}}$

$2I = [x^3]_{-2}^{2} = 8-(-8)$

$I = 8$

R → 1

$f(x) = ax^2+bx$ (or) $f(x) = ax$

because $f(0)=0$

now

$\int_0^1 f(x)\,dx = 1$

$\left[\frac{ax^3}{3}+\frac{bx^2}{2}\right]_0^1 = 1$

$\frac{a}{3}+\frac{b}{2} = 1$

$a=3$ & $b=0$ (or) $a=0$ & $b=2$

then $a=2$

so P → 2

Q. 58. (A)

	List I	List II
P.	Let $y(x) = \cos(3\cos^{-1}x), x \in [-1,1], x \neq \pm\frac{\sqrt{3}}{2}$. Then $\frac{1}{y(x)}\left\{(x^2-1)\frac{d^2y(x)}{dx^2} + x\frac{dy(x)}{dx}\right\}$ equals	**1.** 1
Q.	Let $A_1, A_2, \ldots, A_n$ $(n>2)$ be the vertices of a regular polygon of n sides with its centre at the origin. Let $\overrightarrow{a_k}$ be the position vector of the point A_k, $k = 1,2,\ldots,n$. If $\left\lvert\sum_{k=1}^{n-1}(\overrightarrow{a_k}\times\overrightarrow{a_{k+1}})\right\rvert = \left\lvert\sum_{k=1}^{n-1}(\overrightarrow{a_k}\cdot\overrightarrow{a_{k+1}})\right\rvert$, then the minimum value of n is	**2.** 2
R.	If the normal from the point $P(h,1)$ on the ellipse $\frac{x^2}{6}+\frac{y^2}{3}=1$ is perpendicular to the line $x+y=8$, then the value of h is	**3.** 8
S.	Number of positive solutions satisfying the equation $\tan^{-1}\left(\frac{1}{2x+1}\right)+\tan^{-1}\left(\frac{1}{4x+1}\right)=\tan^{-1}\left(\frac{2}{x^2}\right)$ is	**4.** 9

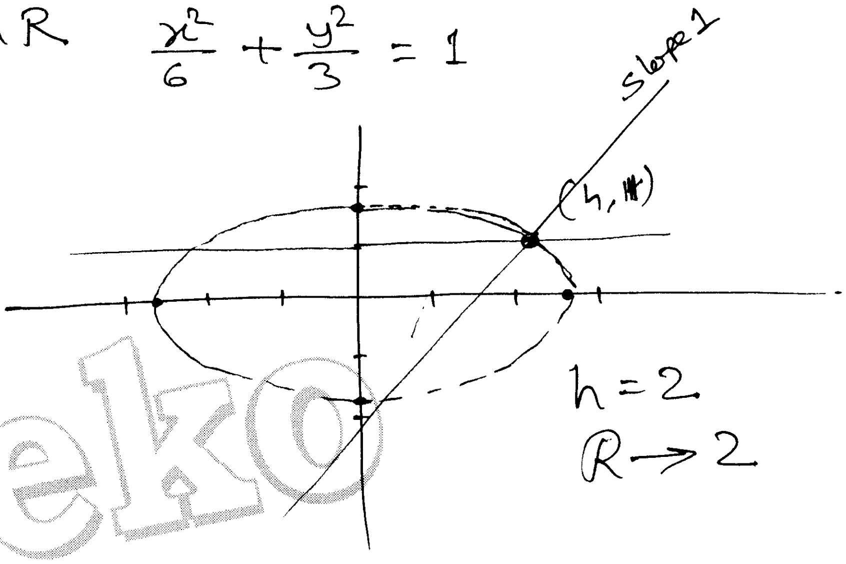
	P	Q	R	S
(A) ✓	4	3	2	1
(B)	2	4	3	1
(C)	4	3	1	2
(D)	2	4	1	3

P.T.O

Sol. 58. Here start with R $\frac{x^2}{6}+\frac{y^2}{3}=1$

Ans Ⓐ

Only Option A is contain R → 2



h = 2

R → 2

Q 59. Let $f_1:\mathbb{R}\to\mathbb{R}$, $f_2:[0,\infty)\to\mathbb{R}$, $f_3:\mathbb{R}\to\mathbb{R}$ and $f_4:\mathbb{R}\to[0,\infty)$ be defined by

Ⓓ

$$f_1(x)=\begin{cases}|x| & \text{if } x<0,\\ e^x & \text{if } x\ge 0;\end{cases}$$

$$f_2(x)=x^2;$$

$$f_3(x)=\begin{cases}\sin x & \text{if } x<0,\\ x & \text{if } x\ge 0\end{cases}$$

and

$$f_4(x)=\begin{cases}f_2(f_1(x)) & \text{if } x<0,\\ f_2(f_1(x))-1 & \text{if } x\ge 0.\end{cases}$$

$f_4(x)$

	List I		List II
P.	f_4 is	**1.**	onto but not one-one
Q.	f_3 is	**2.**	neither continuous nor one-one
R.	$f_2 o f_1$ is	**3.**	differentiable but not one-one
S.	f_2 is	**4.**	continuous and one-one

	P	Q	R	S
~~(A)~~	3	1	4	2 ✓
(B)	1	3	4	2
~~(C)~~	3	1	2	4 ✗
(D)	1	3	2	4

$f_2 \circ f_1 \begin{cases}(|x|)^2 = x^2\\ (e^x)^2\ e^{2x}\end{cases}$

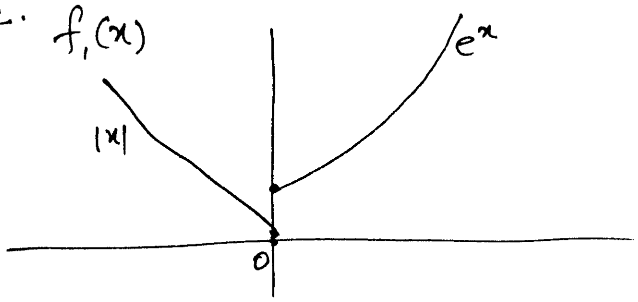
÷(2) 2x

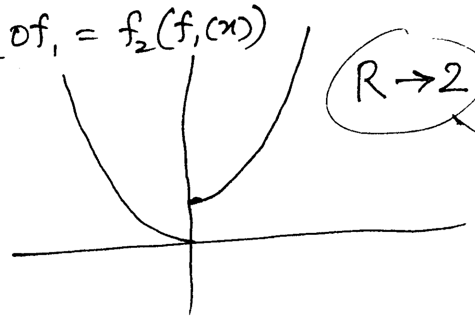
2x

P.T.O.

Sol. 59.
(D)

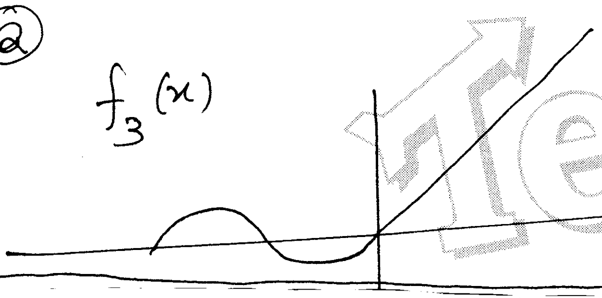
(R)





R→2

(Q)



Q→3

So Ans is (D)

60. Let $z_k = \cos\left(\frac{2k\pi}{10}\right) + i\sin\left(\frac{2k\pi}{10}\right)$; $k = 1, 2, \ldots, 9$.

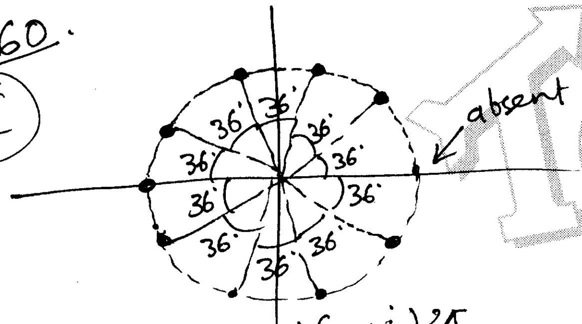
Q (C)

	List I		List II
P.	For each z_k there exists a z_j such that $z_k \cdot z_j = 1$	**1.**	True
Q.	There exists a $k \in \{1, 2, \ldots, 9\}$ such that $z_1 \cdot z = z_k$ has no solution z in the set of complex numbers.	**2.**	False
R.	$\frac{\lvert 1-z_1\rvert\lvert 1-z_2\rvert\cdots\lvert 1-z_9\rvert}{10}$ equals	**3.**	1
S.	$1 - \sum_{k=1}^{9}\cos\left(\frac{2k\pi}{10}\right)$ equals	**4.**	2

Options for Q 60

	P	Q	R	S
(A)	1	2	4	3
~~(B)~~	2	1	3	4
(C)	1	2	3	4
~~(D)~~	2	1	4	3

Sol. 60.
(C)



$\frac{360°}{10} = 36°$

It is actually roots of eqn
$z^{10} = 1$
Sum of all roots $= 0$
$\sum_{k=1}^{9}\cos\left(\frac{2k\pi}{10}\right) + 1 = 0$
$\star + 1 = 0$
$\star = -1$

So
S→
$1 - \star$
$1 - (-1)$
'2'
S→4

P→ $z_k \cdot z_j = e^{i(k+j)\frac{2\pi}{10}} = \cos\left((k+j)\frac{2\pi}{10}\right) + i\sin\left((k+j)\frac{2\pi}{10}\right)$

So it is true P→1

Only Option C is contain P→1 & S→4.